3. *Nasredinov F. S., Seregin N. P., Seregin P. P., Bondarevskiĭ S. I.* A Mössbauer study of a two-electron acceptor impurity of zinc in silicon // Semiconductors. 2000. V. 34. №. 3. P. 269–271.

4. *Немов С. А., Серегин П. П., Кожанова Ю. В., Троицкая Н. Н., Волков В. П., Серегин Н. П., Шамрай В. Ф.* Изменение электронной плотности при сверхпроводящем фазовом переходе в Nb_3Al // Физика твердого тела. 2004. Т. 46. Вып. 2. С. 228–230.

5. *Seregin N. P., Nasredinov F. S., Bondarevskii S. I., Ermolaev A. V., Seregin P. P.* The charge state of copper impurity atoms in AgCl annealed in vacuum or chlorine // Journal of Physics: Condensed Matter. 2001. V. 13. № 11. P. 2671–2676.

6. *Seregin N. P., Nemov S. A., Irkaev S. M.* Study of zinc impurity atoms in GaP, GaAs, and GaSb 67Ga(67Zn) and 67Cu(67Zn) by emission Mössbauer spectroscopy // Semiconductors. 2002. V. 36. № 9. P. 975–976.

7. *Terukov E. I., Seregin P. P., Marchenko A. V.* Experimental determination of the temperature dependence of the superfluid electron density in high-temperature superconductors // Technical Physics Letters 2007. V. 33. № 5. P. 404–406.

REFERENCES

1. *Bordovskij G. A., Nemov S. A., Marchenko A. V., Seregin P. P., Zajtseva A. V.* Messbaujerovskie U-tsentry kak instrument issledovanija boze-kondesatsii v poluprovodnikah // Fizika i tehnika poluprovodnikov. 2008. T. 42. Vyp. 10. S. 1172–1179.

2. *Bordovskij G. A., Terukov E. I., Marchenko A. V., Seregin P. P.* Identifikatsija dvuhjelektronnyh tsentrov s otritsatel'noj korreljatsionnoj energiej v vysokotemperaturnyh sverhprovodnikah // Fizika tverdogo tela. 2009. T. 51. Vyp. 11. S. 2094–2098.

3. *Nasredinov F. S., Seregin N. P., Seregin P. P., Bondarevskiĭ S. I.* A Mössbauer study of a two-electron acceptor impurity of zinc in silicon // Semiconductors. 2000. V. 34. № 3. P. 269–271.

4. *Nemov S. A., Seregin P. P., Kozhanova Jz. V., Troitskaja N. N., Volkov V. P., Seregin N. P., Shamraj V. F.* Izmenenie elektronnoj plotnosti pri sverhprovodjashchem fazovom perehode v Nb3Al // Fizika tverdogo tela. 2004. T. 46. Vyp. 2. S. 228–230.

5. *Seregin N. P., Nasredinov F. S., Bondarevskii S. I., Ermolaev A. V., Seregin P. P.* The charge state of copper impurity atoms in AgCl annealed in vacuum or chlorine // Journal of Physics: Condensed Matter. 2001. V. 13. № 11. P. 2671–2676.

6. *Seregin N. P., Nemov S. A., Irkaev S. M.* Study of zinc impurity atoms in GaP, GaAs, and GaSb 67Ga(67Zn) and 67Cu(67Zn) by emission Mössbauer spectroscopy // Semiconductors. 2002. V. 36. № 9. P. 975–976.

7. *Terukov E. I., Seregin P. P., Marchenko A. V.* Experimental determination of the temperature dependence of the superfluid electron density in high-temperature superconductors // Technical Physics Letters 2007. V. 33. № 5. P. 404–406.

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Quasiparticles, Spectral Functions, and Kinetic Equation in Quantum Fermi Liquid Theory

Green's function method in Kadanoff-Baym version is used to analyze different ways of determining of quasiparticle energies in a normal quantum system of interacting fermions and to derive equations which determine these energies. The appearing differences for the microscopic and phenomenological approaches to the Fermi liquid theory are discussed. The validity of the Landau-Silin kinetic equation for the quasiparticle distribution function at finite temperature is proved on the basis of a proper approximation to the spectral function.

Keywords: Fermi liquid, Green's function, spectral function, quasiparticle, kinetic equation.

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КВАЗИЧАСТИЦЫ, СПЕКТРАЛЬНЫЕ ФУНКЦИИ И КИНЕТИЧЕСКОЕ УРАВНЕНИЕ В ТЕОРИИ КВАНТОВОЙ ФЕРМИ-ЖИДКОСТИ

С помощью метода функций Грина в варианте Каданова и Бейма анализируются различные способы определения квазичастиц в нормальной квантовой системе взаимодействующих фермионов и выводятся уравнения для определения их энергий. Обсуждаются возникающие различия при микроскопическом и феноменологическом подходах к теории ферми-жидкости. На основе использования правильного приближения для спектральной функции устанавливается справедливость кинетического уравнения Ландау — Силина для функции распределения квазичастиц при конечной температуре.

Ключевые слова: ферми-жидкость, функция Грина, спектральная функция, квазичастица, кинетическое уравнение.

1. *Introduction*

The exclusive successfulness of the phenomenological Landau — Silin theory of normal Fermi liquids in predicting and describing a set of new phenomena, among them the zero sound in ^3He and spin waves in non-ferromagnetic metals, made this theory a subject of investigation on the basis of the fundamental microscopic theory [14]. The most general approach to the problem is based on the real-time Green's-function formalism of Martin and Schwinger, further developed by Kadanoff and Baym [5]. This method is capable of describing in a comprehensive way the equilibrium and non-equilibrium properties of many-body systems at zero and finite temperatures. The Kadanoff-Baym (KB) equations were used in several contributions to this evolving field with applications to nuclear matter [9], to one- and two-band semiconductors [16], to plasma oscillations in an electron gas [11], to kinetic equations for quasiparticles in the case of quickly varying in space and time disturbances [10], etc.

In its original form phenomenological normal Fermi liquid theory deals only with the variations of quasiparticle energies, but not with the energies themselves. But in a series of publications devoted to the magnetic properties of 3d-metals [6] and to the binding energy of nuclear matter [8] the conception of quasiparticles in the spirit of Fermi liquid theory was used and different model expressions for the quasiparticle energies were offered. The results of numerical calculations based on the developed in [6; 8] theories proved to be in a good agreement with the corresponding experimental data despite the fact that the widths of the energy levels were not considered small as it is assumed in the ordinary phenomenological Fermi liquid theory and in the original microscopic approach to the Fermi liquid theory [5].

The purpose of this paper is to analyze on the basis of the KB theory various approaches to the definition of quasiparticles in a Fermi liquid, to prove the validity or the model expressions for the quasiparticle energies used in [6; 8], and to prove the validity of the offered in [1] approximation to the spectral function that takes into account the widths of the energy levels. This demands a detailed analysis of the properties of spectral functions of one-particle states in a many-body system. Finally, we will prove the validity of the Landau-Silin (LS) kinetic equation [14] for the quasiparticle distribution function in the case of finite widths of the energy levels for different ways of introduction of quasiparticles in a normal Fermi liquid.

2. *Main formulas of the Kadanoff-Baym theory*

The KB formalism leads to the following general expression for the one-particle spectral function $a\ (\vec{p},\omega)$ of a system in equilibrium [5]:

$$a\ (\vec{p},\omega) = \frac{\Gamma(\vec{p},\omega)}{(\omega - e(\vec{p},\omega))^2 + \Gamma^2(\vec{p},\omega)/4}, \tag{1}$$

where

$$e(\vec{p},\omega) = E^{HF}(\vec{p}) + \mathrm{Re}\sum\nolimits_c(\vec{p},\omega) \tag{2}$$

and $E^{HF}(\vec{p})$ is a one-particle energy in the Hartree-Fock approximation. Real and imaginary ($\Gamma(\vec{p},\omega)$) parts of the correlation self-energy function $\sum_c(\vec{p},\omega)$ are related to each other through the Hilbert transform:

$$\mathrm{Re}\sum\nolimits_c(\vec{p},\omega) = P\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Gamma(\vec{p},\omega')}{\omega-\omega'}. \tag{3}$$

Here P refers to a principal value integration.

The spectral function (1) satisfies the exact sum rule

$$\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}a\,(\vec{p},\omega) = 1. \tag{4}$$

In the case of slowly varying in space and time disturbances, after the transition to the Wigner coordinates

$$\vec{R} = (\vec{r}_1 + \vec{r}_1')/2\,, \qquad \vec{r} = \vec{r}_1 - \vec{r}_1'\,, \qquad T = (t_1 + t_1')/2\,, \qquad t = t_1 - t_1' \tag{5}$$

and the performance of the Fourier transform with respect to and, all the quantities ntering the theory are considered to be the functions of $\vec{p},\omega,\vec{R},T$. For example,

$$a = a(\vec{p}\,\omega;\vec{R}\,T).$$

If we take into account only the first derivatives with respect to slowly varying quantities $\vec{R}$, and T in the KB equations for the correlation functions, we come to the following equation for the spectral function $a(\vec{p}\,\omega;\vec{R}\,T)$:

$$\left[\omega - e(\vec{p}\,\omega;\vec{R}\,T), a(\vec{p}\,\omega;\vec{R}\,T)\right] + \left[\mathrm{Re}g(\vec{p}\,\omega;\vec{R}\,T), \Gamma(\vec{p}\,\omega;\vec{R}\,T)\right] = 0\,, \tag{6}$$

and to the generalized KB kinetic equation for the correlation function $g^<(\vec{p}\,\omega;\vec{R}\,T)$

$$\left[\omega - e(\vec{p}\,\omega;\vec{R}\,T), g^<(\vec{p}\,\omega;\vec{R}\,T)\right] + \left[\mathrm{Re}g(\vec{p}\,\omega;\vec{R}\,T), \sum\nolimits^<(\vec{p}\,\omega;\vec{R}\,T)\right] = \left(\sum\nolimits^< g^> - \sum\nolimits^> g^<\right)(\vec{p}\,\omega;\vec{R}\,T). \tag{7}$$

Here $[A,B]$ is the generalized Poisson bracket defined by the expression [5]:

$$[A,B] = \frac{\partial A}{\partial\omega}\frac{\partial B}{\partial T} - \frac{\partial A}{\partial T}\frac{\partial B}{\partial\omega} - \frac{\partial A}{\partial\vec{p}}\cdot\frac{\partial B}{\partial\vec{R}} + \frac{\partial A}{\partial\vec{R}}\cdot\frac{\partial B}{\partial\vec{p}}, \tag{8}$$

and E^{HF} and $\mathrm{Re}\sum_c$ include the interaction with the external field. The exact solution of Eq. (6) is given by the expression [5]:

$$g(\vec{p}\,z;\vec{R}\,T) = \left(z - E^{HF}(\vec{p}\,;\vec{R}\,T) - \mathrm{Re}\sum\nolimits_c(\vec{p}\,z;\vec{R}\,T)\right)^{-1}. \tag{9}$$

In fact, the solution (9) results in almost the same evaluation of the spectral function $a(\vec{p}\,\omega;\vec{R}\,T)$ as in the equilibrium state:

$$a(\vec{p}\,\omega;\vec{R}\,T)=\frac{\Gamma(\vec{p}\,\omega;\vec{R}\,T)}{(\omega-e(\vec{p}\,\omega;\vec{R}\,T))^2+\Gamma^2(\vec{p}\,\omega;\vec{R}\,T)/4}. \tag{10}$$

Equation (7) provides an exact description of the response to slowly varying disturbance. All the quantities appearing in this equation may be expressed in terms of correlation and self-energy functions.

The result (10) means that in the case of slowly varying disturbances the approximations for the nonequilibrium spectral function may be written in the same form as in the equilibrium case.

3. *Quasiparticles in a normal Fermi liquid*

In the KB formalism for a system in equilibrium the quasiparticle energy $E=E(\vec{p})$ is defined as a solution of the equation [5]:

$$E(\vec{p})=e(\vec{p},\omega)\big|_{\omega=E(\vec{p})}. \tag{11}$$

Now we expand $e(\vec{p},\omega)$ as a function of ω in Taylor series near the value $E(\vec{p})$ and save only linear terms:

$$e(\vec{p},\omega)=E(\vec{p})+\left.\frac{\partial e(\vec{p},\omega)}{\partial\omega}\right|_{\omega=E(\vec{p})}(\omega-E(\vec{p})). \tag{12}$$

We substitute (12) into Eq. (1) and get the formula for a_{QP} called the spectral function of the quasiparticle state:

$$a_{QP}\,(\vec{p},\omega)=\frac{Z^2\Gamma(\vec{p},E(\vec{p}))}{(\omega-E(\vec{p}))^2+Z^2\Gamma^2(\vec{p},E(\vec{p}))/4}, \tag{13}$$

where $Z=Z(\vec{p})$ is a renormalizing factor defined by the expression

$$Z^{-1}=1-\left.\frac{\partial e(\vec{p},\omega)}{\partial\omega}\right|_{\omega=E(\vec{p})}. \tag{14}$$

It is easy to prove that $Z<1$ for all values of $\vec{p}$ when correlation energy $\sum_c$ is taken into account [5]. The quantity $\gamma=Z\Gamma$ can be considered the width of the quasiparticle state, and Eq. (13) for the spectral function may be rewritten as

$$a_{QP}\,(\vec{p},\omega)=Z\frac{\gamma}{(\omega-E(\vec{p}))^2+\gamma^2/4}. \tag{15}$$

A severe drawback with a_{QP} is that it normalizes to Z rather than to 1 as in Eq. (4):

$$\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,a_{QP}\,(\vec{p},\omega)=Z<1. \tag{16}$$

If we neglect the width of the quasiparticle state $\gamma\to 0$, Eq. (15) reads:

$$a_{QP}\ (\vec{p},\omega) = 2\pi Z\delta(\omega - E\ (\vec{p})) \tag{17}$$

with the same normalization determined by Eq. (16).

We will consider a system at zero temperature with a spherical Fermi-surface and will use a separable form for the widths of the energy levels:

$$\Gamma(\vec{p},\omega) = \alpha(p)\sqrt{|\omega - \mu|}. \tag{18}$$

Such parameterization of the energy level's width allows to describe many realistic features of quantum many-body systems [15]. The model expression (18) provides the convergence of the integral in the Hilbert transform (3). The quantity μ in Eq. (18) is a chemical potential of a system under consideration. In this case Eq. (11) for the quasiparticle energy E with a help of Eqs. (2) and (3) may be written in the form:

$$E = E^{HF}(p) + \alpha(p)\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\sqrt{|\omega - \mu|}}{E - \omega}. \tag{19}$$

The integral I in Eq. (19) may be evaluated with a help of the residue method or after some transformations may be represented as

$$I = \frac{E - \mu}{\pi}\int_{0}^{\infty}\frac{\sqrt{\omega}d\omega}{(E - \mu)^2 - \omega^2}, \tag{20}$$

and may be evaluated with a help of the formula [4]:

$$\int_{0}^{\infty}\frac{x^{m-1}dx}{1 - x^n} = \frac{\pi}{n}ctg\frac{\pi m}{n}, \quad m < n, \tag{21}$$

where *m* and *n* are rational numbers. Finally we come to the following equation for the quasiparticle energy $E(p)$:

$$E(p) = E^{HF}(p) + \alpha(p)\sqrt{|E(p) - \mu|}\,\mathrm{sgn}(E(p) - \mu), \tag{22}$$

which determines the values of the energies of quasiparticles $(E(p) - \mu > 0)$ and quasiholes $(E(p) - \mu < 0)$. In normal Fermi liquids at zero temperature the energy of a quasiparticle on the Fermi level equals the energy in the Hartree-Fock approximation and the damping of the energy levels is absent [5].

Quasiparticle energy can be defined in a way different from Eq. (11). For example, we may introduce a quasiparticle energy $E_1 = E_1(p)$ as a solution of the equation

$$E_1 = E^{HF}(p) + \alpha(p)\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\sqrt{|\omega - \mu|}}{e(p,\omega) - \omega}. \tag{23}$$

In accordance with Eq. (12), we use the approximation

$$e(p,\omega) = E_1(p) + \left.\frac{\partial e(\vec{p},\omega)}{\partial \omega}\right|_{\omega=E_1(\vec{p})} (\omega - E_1(\vec{p})). \tag{24}$$

In this approximation we get the equation (23) for $E_1(p)$ in the form

$$E_1 = E^{HF}(p) + Z_1\alpha(p)\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\sqrt{|\omega-\mu|}}{E_1-\omega}, \tag{25}$$

where the renormalizing factor $Z_1 = Z_1(p)$ is defined by the expression

$$Z_1^{-1} = 1 - \left.\frac{\partial e(\vec{p},\omega)}{\partial \omega}\right|_{\omega=E_1(\vec{p})}. \tag{26}$$

Comparing the expressions (19) and (25) it is easy to see that the equation (25) for E_1 differs from Eq. (22) for E only by the factor Z_1 before the second term in the right side of Eq. (22):

$$E_1(p) = E^{HF}(p) + Z_1\alpha(p)\sqrt{|E_1(p)-\mu|}\,\mathrm{sgn}(E_1(p)-\mu). \tag{27}$$

Eqs. (22) and (25) mean that in the case of the separable model (18) for the widths of the energy levels quasiparticle energy may be represented as a renormalized one-particle energy in the Hartree-Fock approximation. The energy $E_1(p)$ is closer to the value of $E^{HF}(p)$ than $E(p)$, but it makes no difference for the phenomenological theory when renormalization is described in terms of some phenomenological parameters. In the case of a microscopic theory of a Fermi liquid the difference in the definitions of quasiparticle energies can be essential. Square root in Eq. (18) may be changed for some other expression providing the convergence of the integral in the Hilbert transform (3), but in this case probably it will be impossible to find solutions of the equations for the quasiparticle energy in an analytic form.

4. *Spectral function and kinetic equation*

Different approximations to the spectral function (1) linear in the energy level width Γ were considered in the series of publications [7; 13; 17] in order to describe realistic features of quantum system of interacting particles in equilibrium and nonequilibrium states.. One of the goals was to prove the validity of the LS kinetic equation for a normal Fermi liquid at finite temperature. All approximations offered in [7; 13; 17] and in some other publications don't satisfy Eq. (6) and thus they are not adequate for a proper description of nonequilibrium phenomena. In particular, all these works failed to prove the validity of the Landau-Silin equation at finite temperature in a lawful mathematical way. It also means that the approximations to the spectral function offered in [7; 13; 17] are not adequate for a description of equilibrium phenomena due to genetic links between the expressions (1) and (10).

The approximation that satisfies Eq. (6) was offered in [1]:

$$a_i(\vec{p},\omega) = 2\pi Z\delta(\omega - E(\vec{p})) + Z^2 P\frac{\Gamma(\vec{p},\omega)}{(\omega - E(\vec{p}))^2}. \tag{28}$$

The validity of the (LS) kinetic equation was proved in [5] on the basis of microscopic theory only for zero temperature. The necessity to prove its validity at finite temperature was determined by the experimental discovery of superfluid transition in ^3He at a temperature lower than the temperature at which the phenomena predicted and described by the normal Fermi liquid theory were observed. The approximation (28) for the spectral function is the basis of such a proof with the precision to the linear in Γ terms.

The approximation (28) can be obtained on the basis of the following relation of the Fourier transform theory [1]:

$$\int_{\infty}^{\infty} dt \exp(-\frac{\Gamma}{2}|t|) \exp(itx) = \frac{\Gamma}{x^2 + \Gamma^2/4}\ ,\ \Gamma > 0. \tag{29}$$

The expansion of the first exponent in the left side of (29) in Taylor series in powers of Γ with the subsequent term- by-term integration with a help of the formulas for the Fourier transforms of powers of $|t|$ [3] allow to generalize the results of [1] and to get the expansion of the spectral function $a\ (\vec{p},\omega)$ in powers of Γ. It can be proved that the sum of the terms with odd powers of Γ gives the complete expression (1) for the spectral function. The series of the terms with odd powers of Γ may be useful for the evaluation of different equilibrium properties of a Fermi liquid.

Each term with even power of the width of the energy level in this expansion contains a Dirac delta function. The sum of the terms with even powers of Γ can be represented in the form:

$$a_{even}(\vec{p},\omega) = 2\pi \frac{(\omega - e(\vec{p},\omega))^2}{(\omega - e(\vec{p},\omega))^2 + \Gamma^2(\vec{p},\omega)/4} \delta(\omega - e(\vec{p},\omega)). \tag{30}$$

Due to the presence of delta function in the numerator it is obvious that the contributions of each term and of the whole sum (30) of the terms with even powers of Γ to the sum rule (4) and to all formulas for different physical properties of a Fermi liquid equals zero. These statements generalize the results obtained in (9).

Main problems in deriving the LS kinetic equation at finite temperature are associated with the necessity of the elimination of the second Poisson bracket in the left side of Eq. (7) in a mathematically lawful way. The reason of the failure of the attempts of such elimination in [13,17] was the usage of improper approximations to the spectral function. A special name "puzzling term" for the second Poisson brackets in Eqs. (6) and (7) was offered in [17] after such useless attempts. The proof of the validity of the LS kinetic equation may be produced in the following way. The first term in the expression (28) for the spectral function after the substitution to the first Poisson bracket in Eq. (7) leads to the LS kinetic equation [1]:

$$\frac{\partial n}{\partial T} + \frac{\partial E}{\partial \vec{p}} \cdot \frac{\partial n}{\partial \vec{R}} - \frac{\partial E}{\partial \vec{R}} \cdot \frac{\partial n}{\partial \vec{p}} = I_{collision}, \tag{31}$$

where n is the quasiparticle distribution function:

$$n(\vec{p};\vec{R}T) = f(\vec{p}\,\omega;\vec{R}\,T)\Big|_{\omega = E(\vec{p};\vec{R}T)}. \tag{32}$$

The second term in the expression (28) after the substitution to the first Poisson bracket in Eq. (7) cancels the "puzzling term" with the second Poisson bracket in this equation after the sub-

stitution of the expression (9) written with the precision to the linear in Γ terms. Right in the same way it may be proved that the approximation (28) satisfies Eq. (6).

5. *Conclusions*

Fermi liquid theory, and more generally, a concept of quasiparticles, stands out as one of the high points of modern theoretical physics, a theory whose profundity goes beyond mere phenomenology [2; 12]. The major emphasis in the investigations in this field is on the practical development and application of the theory and on the description of its microscopic derivation. There is no way of knowing a priory whether or not a given system of fermions is a normal Fermi liquid; the only way to decide this is to measure the properties of the system experimentally and see if the results are consistent with the predictions of the theory. The results of our work allow to make the following conclusions.

The quasiparticle energy, at least in the case of separable models for a width of energy levels may be represented as a renormalized energy in the Hartree-Fock approximation. This proves the validity of the model expressions for the quasiparticle energies used in [6; 8] and explains the agreement of the produced there numerical calculations with the experimental data.

Different ways of the definition of quasiparticles may be essential only in the frame of microscopic approach to a Fermi liquid theory, but they lead to the same expressions for the spectral functions, and as a consequence, to the same form of the phenomenological theory.

The Landau-Silin kinetic equation for a normal Fermi liquid is valid at finite temperature with the precision to the linear terms in the width of one-particle energy levels. Due to the remarks after the formula (30) it can be considered that the LS kinetic equation is valid at finite temperature with the precision to the quadratic terms in the width of one-particle energies.

REFERENCES

1. *Arshad M., Kondratyev A. S.*, and *Siddique I.* Spectral function and kinetic equation for a normal Fermi liquid // Phys. Rev. B. 2007. Vol. 76. P. 054306–7.

2. *Baym G., Pethick C.* Landau Fermi Liquid Theory. Weinheim: WILEY-VCH Verlag GmbH and Co, 2004. 203 p.

3. *Brychkov Yu. A., Prudnikov A. P.* Integral Transform of Generalized Functions. N.Y.: Gordon and Breach, 1989.

4. *Gradshtein I. S., Ryzhick I. M.* Tables of integrals, sums, series and products. M.: Fismatlit, 1963. 1100 p.

5. *Kadanoff L. P., Baym G.* Quantum Statistical Mechanics. Cambridge, Massachusetts: Perseus Books, 1989. 203 p.

6. *Kazansky A. K., Kondratyev A. S.,* and *Uzdin V. M.* The description of magneto-ordered layer-nonuniform state in a periodic Anderson's model // ETP. 1988. Vol. 94. No. 2. P. 274–283.

7. *Kohler H. S.* Spectral functions in nuclear matter // Phys. Rev. C. 1992. Vol. 46. No. 5. P. 1687–1696.

8. *Kohler H. S., Malfliet R.* Extended quasiparticle approximation and Brueckner theory // Phys. Rev. C. 1993. Vol. 48. No. 3. P. 1034–1045.

9. *Kohler H. S., Morawetz K*. Correlations in many-body systems with two-time Green's functions // Phys. Rev. C. 2001. Vol. 64. P. 024613–13.

10. *Kondratyev A. S., Shahid N.* Different forms of the Kadanoff-Baym equations in quantum statistical mechanics // Low Temp. Phys. 2011. Vol. 37. No. 9/10. P. 977–981.

11. *Kwong N.-H., Bonitz M*. Real-Time Kadanoff-Baym Approach to Plasma Oscillations in a Correlated Electron Gas // Phys. Rev. Lett. 2000. Vol. 84. No. 8. P. 1768–1771.

12. *Migdal A. B*. Theory of Finite Fermi Systems and Applications to Atomic Nuclei. N.Y.: Interscience, 1962. 287 p.

13. *Morawetz K., Roepke G.* Memory effects and virial corrections in nonequilibrium dense systems // Phys. Rev. E. 1995. Vol. 51. No. 5. P. 4246–4261.

14. *Nozieres P., Pines D.* The Theory of Quantum Liquids. Normal Fermi Liquids. V. 1. Cambridge, Massachusetts: Perseus Books, 1999. 355 p.

15. *Reggiani L., Lugli P.,* and *Jauho A. P.* Quantum kinetic equation for electronic transport in nondegenerate semiconductors // Phys. Rev. B. 1987. Vol. 36. No. 12. P. 6602–6608.

16. *Schmielau T.* Phase Space Filling in the Electron-Hole Pair Green's Function: Proc. of the Conf. "Progress in Nonequilibrium Green's Functions". Dresden, Germany, 2002. P. 265–273.

17. *Spicka V., Lipavsky P.* Quasiparticle Boltzman equation in semiconductors // Phys. Rev. B. 1995. Vol. 52. No. 20. P. 14615–14635.

А. В. Ляпцев

СИММЕТРИЯ РЕГУЛЯРНЫХ И ХАОТИЧЕСКИХ ДВИЖЕНИЙ В ЗАДАЧАХ НЕЛИНЕЙНОЙ ДИНАМИКИ. УРАВНЕНИЕ ДУФФИНГА

Предложен метод исследования симметрии регулярных и хаотических движений в задачах нелинейной динамики. При регулярных движениях исследуется симметрия фазовой траектории системы. При хаотических движениях симметрия исследуется при помощи сечений Пуанкаре. Показано, что для нелинейного осциллятора, описываемого уравнением Дуффинга, группа симметрии уравнений в зависимости от параметров изоморфна различным точечным группам. При этом симметрия решений может быть такой же, как симметрия исходной группы или нарушаться в зависимости от параметров задачи.

Ключевые слова: нелинейная динамика, динамический хаос, сечения Пуанкаре, странный аттрактор.

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The Symmetry of Regular and Chaotic Motions in Nonlinear Dynamic Problems. Duffing Equation.

A method of investigation of symmetry of regular and chaotic motions in nonlinear dynamic problems is suggested. The symmetry of regular motions is investigated by means of analysis of phase trajectory. The symmetry of chaotic motions is investigated by means of analysis of Poincare cross-sections. It is shown that for a nonlinear oscillator described by Duffing equation, group symmetry of the equations depending on a parameter is isomorphic to a variety of point groups. At the same time the symmetry of the solutions might be the same as the symmetry of the initial group or violated depending on the parameters of the problem.

Keywords: nonlinear dynamics, dynamic chaos, the Poincare cross-section, of a strange attractor.

Симметрия играет значительную роль как в задачах классической механики, так и в задачах квантовой механики. Существенным различием является то, что в квантовой механике после установления равновесия плотность вероятности является полносимметричной по отношению к группе симметрии исходных уравнений, в то время как в классической физике решения могут не быть полносимметричными, то есть симметрия может нарушаться. Чтобы это понять, достаточно обратиться к решениям кеплеровской задачи в квантовой (атом водорода) и классической (движение планет) задачах.